Corrigé

I) 1

a) $E(0,y,z) \in d \iff \begin{cases} -y+z=2 \\ y+z=0 \end{cases} \iff \begin{cases} z=1 \\ y=-1 \end{cases}$ $\quad E(0,-1,1)$

b) $F(x,1,1) \in d \iff \begin{cases} x-1+1=2 \\ 2x+2=0 \end{cases}$

$\iff \begin{cases} x=2 \\ x=-1 \end{cases}$ un tel point F n'existe pas

2 $\vec{AB}(-1,2,4)$

$M(x,y,z) \in AB \iff \boxed{\begin{cases} x=1-\lambda & (1) \\ y=2\lambda & (2) \\ z=-1+4\lambda & (3) \end{cases} \quad \lambda \in \mathbb{R}}$ système d'éq. paramétriques de AB

de (2) : $\lambda = \frac{y}{2}$

ds (1) : $x = 1 - \frac{y}{2} \iff$

ds (3) : $z = -1 + 2y \iff$ $\boxed{\begin{cases} 2x+y-2=0 \\ 2y-z-1=0 \end{cases}}$ système d'éq. cartésiennes de AB

3 $\begin{cases} y+3z=1 \\ 2x-y+z=3 \\ x-y+2z=-1 \end{cases} \overset{E_1 \leftrightarrow E_2}{\iff} \begin{cases} 2x-y+z=3 \\ y+3z=1 \\ x-y+2z=-1 \end{cases}$

$\underset{E_3 | E_1 - 2E_3}{\iff} \begin{cases} 2x-y+z=3 \\ y+3z=1 \\ y-3z=5 \end{cases}$

$\underset{E_3 | E_2 + E_3}{\iff} \begin{cases} 2x-y+z=3 \\ y+3z=1 \\ 2y=6 \end{cases}$

$\iff \begin{cases} 2x=6-z \\ z=\frac{1-3}{3} \\ y=3 \end{cases} \iff \begin{cases} x=\frac{10}{3} \\ y=3 \\ z=-\frac{2}{3} \end{cases}$

$\boxed{S = \left\{ \left(\frac{10}{3}, 3, -\frac{2}{3} \right) \right\}}$

II) 1) $C_4^3 + C_5^3 + C_6^3 = 4 + \frac{5\cdot 4}{2} + \frac{6\cdot 5\cdot 4}{6}$

$= 4 + 10 + 20 = 34$

2) $5\cdot 5\cdot 4 = 100$

3) $\underbrace{9\cdot 6\cdot 13}_{}$ (1ère boule non verte) $+$ $\underbrace{6\cdot 5\cdot 13}_{}$ (1ère boule verte)

$= 6\cdot 13\,(9+5)$

$= 78\cdot 14 = 1092$

$(4+2+4=10)$

III) ① $A = \ln 2 - \ln e^4 + \ln e + \ln\sqrt{2} - \ln\sqrt{8} + \ln e^2$

$= \ln 2 - 4 + 1 + \frac{1}{2}\ln 2 - \frac{3}{2}\ln 2 + 2$

$= -1$

② C.E : $x \neq \frac{1}{2}$

$\text{dom} f = \mathbb{R} - \left\{\frac{1}{2}\right\}$

$f'(x) = \dfrac{(2x-1)\cdot e^x - e^x\cdot 2}{(2x-1)^2}$

$= \dfrac{2xe^x - 3e^x}{(2x-1)^2} = \dfrac{e^x(2x-3)}{(2x-1)^2}$

③ $e^{x^2-8} - e^{2x} \geqslant 0$

$e^{x^2-8} \geqslant e^{2x}$

$x^2 - 8 \geqslant 2x$

$x^2 - 2x - 8 \geqslant 0$

$\Delta = 4 + 32 = 36$

$x_{1,2} = \dfrac{2 \pm 6}{2}$ ⟨ $x_1 = 4$, $x_2 = -2$

$\boxed{S =]-\infty, -2] \cup [4, +\infty[}$

$(3+4+5 = 12\text{ pts})$

IV

① $I = \int_0^1 \frac{3}{(x+1)^2}\,dx = -3\left[\frac{1}{x+1}\right]_0^1$

$= -3\left(\frac{1}{2} - 1\right) = \boxed{\frac{3}{2}}$

② $A(x) = \int x^2 \ln x \, dx$ I.p.p $\begin{cases} f(x) = \ln x \\ g'(x) = x^2 \end{cases}$ $\begin{cases} f'(x) = \frac{1}{x} \\ g(x) = \frac{1}{3}x^3 \end{cases}$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$

$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + k\ ,\ k \in \mathbb{R}}$

$B(x) = \int \frac{e^{-x}}{1+e^{-x}}\,dx$ poser : $u(x) = 1 + e^{-x}$

$u'(x) = -e^{-x}$

$= \boxed{-\ln(1+e^{-x}) + k,\ k \in \mathbb{R}}$

$(3+5+4 = 12$ pts$)$

V

① C.E : $x+3>0 \iff x>-3$

$\text{dom} f =]-3, +\infty[$

② $F'(x) = 1 \cdot \ln(x+3) + (x+3)\cdot\frac{1}{x+3} - 1$

$= \ln(x+3) = f(x)$

donc F est une primitive de f

③ $\mathcal{A} = \int_{-2}^{2} \left[\ln(x+3) - (0{,}25x + 0{,}5)\right] dx$

$= \left[(x+3)\ln(x+3) - x - 0{,}25\frac{x^2}{2} - 0{,}5x\right]_{-2}^{2}$

$= 5\ln 5 - 2 - 0{,}5 - 1 - \left(\ln 1 + 2 - 0{,}5 + 1\right)$

$= 5\ln 5 - 3{,}5 - 2{,}5$

$= 5\ln 5 - 6$

$\simeq 2{,}05$ u.a.

$(1+3+7 = 11$ pts$)$
