Corrigé (E, F, G, juin 2010)

I) 1) a) $A(1,-1,0)$ et $B(3,0,-1)$

$M(x,y,z) \in d \Leftrightarrow \exists k \in \mathbb{R}$ tel que $\overrightarrow{AM} = k \cdot \overrightarrow{AB}$

$\Leftrightarrow \exists k \in \mathbb{R}$ tel que $\begin{cases} x-1 = 2k & (1) \\ y+1 = k & (2) \\ z = -k & (3) \end{cases}$

(2) dans (1) et (3) : $\begin{cases} x-1 = 2(y+1) \\ z = -(y+1) \end{cases} \Leftrightarrow \begin{cases} x-2y-3=0 \\ y+z+1=0 \end{cases} \leftarrow$ système d'équations cartésiennes de d

b) $P(1,1,1)$, $Q(3,2,0)$, $R(0,-\frac{5}{2},0)$

$M(x,y,z) \in \Pi \Leftrightarrow \exists r,s \in \mathbb{R}$ tels que $\overrightarrow{PM} = r \cdot \overrightarrow{PQ} + s \cdot \overrightarrow{PR}$

$\Leftrightarrow \exists r,s \in \mathbb{R}$ tels que $\begin{cases} x-1 = 2r-s \\ y-1 = r-\frac{7}{2}s \\ z-1 = -r-s \end{cases}$

$$\begin{cases} x-1 = 2r-s \\ y-1 = r-\frac{7}{2}s \\ z-1 = -r-s \end{cases}$$

$\Updownarrow$ $(E_2) \mid (E_1)-2(E_2)$; $(E_3) \mid (E_2)+(E_3)$

$$\begin{cases} x-1 = 2r-s \\ x-2y+1 = 6s \\ y+z-2 = -\frac{9}{2}s \end{cases}$$

$\Updownarrow$ $(E_3) \mid \frac{4}{3}(E_3)+(E_2)$

$$\begin{cases} x-1 = 2r-s \\ x-2y+1 = 6s \\ x-\frac{2}{3}y+\frac{4}{3}z-\frac{5}{3}=0 \end{cases}$$

Equation cartésienne de Π :

$x-\frac{2}{3}y+\frac{4}{3}z-\frac{5}{3}=0 \quad | \cdot 3$

$3x-2y+4z-5=0$

c) Résolvons le système suivant :

$$\begin{cases} x-2y-3=0 \\ y+z+1=0 \\ 3x-2y+4z-5=0 \end{cases}$$

$\Updownarrow$

$$\begin{cases} x = 2y+3 \\ z = -y-1 \\ 3(2y+3)-2y+4\cdot(-y-1)-5=0 \end{cases}$$

$\Updownarrow$

$$\begin{cases} x = 2y+3 \\ z = -y-1 \\ 0y+0=0 \end{cases}$$

$S = \{(2y+3,\ y,\ -y-1),\ y \in \mathbb{R}\}$

Ainsi $d \cap \Pi = d$

2) $d' \equiv \begin{cases} 2x+y-1=0 \\ -x+z-3=0 \end{cases} \Leftrightarrow \begin{cases} y=-2x+1 \\ z=x+3 \end{cases}$

p.ex. prenons $x=0$, alors $y=1$ et $z=3$ et $F(0,1,3) \in d'$

prenons $x=1$, alors $y=-1$ et $z=4$ et $G(1,-1,4) \in d'$

donc $\overrightarrow{FG}(1,-2,1)$ est un vecteur directeur de d'.

II) a) $\left(e^{3-x}\right)^2 = \frac{1}{e^{x-2}}$

$\Leftrightarrow e^{6-2x} = e^{2-x}$

$\Leftrightarrow 6-2x = 2-x$

$\Leftrightarrow x = 4$

$S = \{4\}$

b) $2\cdot \ln(3-x) - \ln(2x-4) \le \ln\left(\frac{x+1}{2}\right)$ (I)

CE : 1) $3-x > 0 \Leftrightarrow x < 3$

2) $2x-4 > 0 \Leftrightarrow x > 2$

3) $\frac{x+1}{2} > 0 \Leftrightarrow x > -1$

$D =]2;3[$

(I) $\Leftrightarrow \ln(3-x)^2 \le \ln(2x-4) + \ln\left(\frac{x+1}{2}\right)$

$\Leftrightarrow \ln(3-x)^2 \le \ln\left[(2x-4)\cdot\frac{x+1}{2}\right]$

$\Leftrightarrow (3-x)^2 \le (2x-4)\cdot\frac{x+1}{2}$

$\Leftrightarrow 9-6x+x^2 \le (x-2)\cdot\cancel{2}\cdot\frac{x+1}{\cancel{2}}$

$\Leftrightarrow 9-6x+\cancel{x^2} \le \cancel{x^2}+x-2x-2$

$\Leftrightarrow 5x \ge 11$

$\Leftrightarrow x \ge \frac{11}{5}$

$S = \left[\frac{11}{5};+\infty\right[\cap D = \left[\frac{11}{5};3\right[$

III) a) $f(x) = 2\cdot\ln\frac{1-3x}{x+2}$

CE: $\frac{1-3x}{x+2} > 0$ et $x+2 \neq 0$

x	$-\infty$		-2		$\frac{1}{3}$		$+\infty$
$1-3x$		$+$		$+$	0	$-$	
$x+2$		$-$	0	$+$		$+$	
$\frac{1-3x}{x+2}$		$-$	$\Vert$	$+$	0	$-$	

dom $f = \left]-2;\frac{1}{3}\right[$

$\forall x \in \left]-2;\frac{1}{3}\right[\quad f'(x) = 2\cdot\frac{\left(\frac{1-3x}{x+2}\right)'}{\left(\frac{1-3x}{x+2}\right)}$

or $\left(\frac{1-3x}{x+2}\right)' = \frac{(1-3x)'\cdot(x+2)-(1-3x)\cdot(x+2)'}{(x+2)^2} = \frac{-3(x+2)-(1-3x)}{(x+2)^2} = \frac{-7}{(x+2)^2}$

et $f'(x) = 2\cdot\frac{-7}{(x+2)^2}\cdot\frac{x+2}{1-3x} = \frac{-14}{(x+2)(1-3x)}$

b) $\int_e^{e^2}\frac{1}{x\ln^2 x}\,dx = \int_e^{e^2}\frac{1}{x}\cdot\ln^{-2}x\,dx$

posons $u(x) = \ln x$

$u'(x) = \frac{1}{x}$

$= \int_e^{e^2} u'(x)\,u^{-2}(x)\,dx = \left[\frac{u^{-1}(x)}{-1}\right]_e^{e^2} = \left[-\frac{1}{u(x)}\right]_e^{e^2} = \left[-\frac{1}{\ln x}\right]_e^{e^2}$

$= -\frac{1}{\ln e^2} + \frac{1}{\ln e} = -\frac{1}{2} + 1 = \frac{1}{2}$

c) $\int_0^1 (1-2x)e^x\,dx$ posons $u(x) = 1-2x$; $u'(x) = -2$

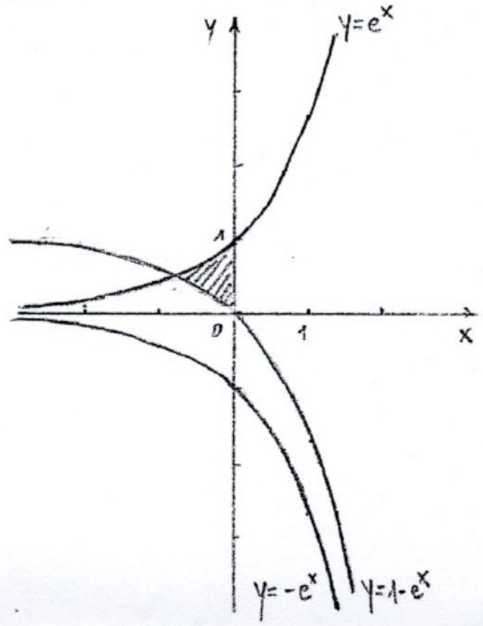
$v'(x) = e^x$; $v(x) = e^x$

$= \left[(1-2x)e^x\right]_0^1 + \int_0^1 2e^x\,dx = \left[e^x - 2xe^x + 2e^x\right]_0^1 = \left[3e^x - 2xe^x\right]_0^1$

$= 3e - 2e - (3-0) = e-3$

IV) a)

x	-2	-1	0	1	$1{,}5$
$f(x)$	$0{,}1$	$0{,}4$	1	$2{,}7$	$4{,}5$



b) $y = e^x$

$\downarrow$ symétrie p.r. à l'axe Ox

$y = -e^x$

$\downarrow$ translation de vecteur $\vec{u}(0,1)$

$y = 1-e^x$

c) $f(x) = g(x) \Leftrightarrow e^x = 1-e^x$

$\Leftrightarrow 2e^x = 1 \Leftrightarrow e^x = \frac{1}{2} \Leftrightarrow x = \ln\frac{1}{2} \simeq -0{,}7$

d) $A = \int_{\ln\frac{1}{2}}^{0} (f(x)-g(x))\,dx$

$= \int_{\ln\frac{1}{2}}^{0} (e^x - 1 + e^x)\,dx = \int_{\ln\frac{1}{2}}^{0} (2e^x - 1)\,dx$

$= \left[2e^x - x\right]_{\ln\frac{1}{2}}^{0} = 2 - 0 - \left(2\cdot\frac{1}{2} - \ln\frac{1}{2}\right)$

$= 2 - 1 + \ln\frac{1}{2} = 1 - \ln 2 \simeq 0{,}31$ u. d'aire

V) 1) a) $C_{32}^5 = \frac{32!}{5!\,27!} = 201.376$

b) $C_4^1 \cdot C_4^2 \cdot C_{24}^2 = 4 \cdot \frac{4!}{2!\,2!} \cdot \frac{24!}{2!\,22!} = 4 \cdot 6 \cdot 276 = 6624$

2) a) $A_{32}^5 = \frac{32!}{27!} = 24.165.120$

b) $4 \cdot C_8^2 \cdot 8^3 \cdot 5! = 6\,881\,280$

c) $A_{32}^5 - A_{28}^5 = \frac{32!}{27!} - \frac{28!}{23!} = 24.165.120 - 11.793.600 = 12.371.520$