

EXAMEN DE FIN D'ÉTUDES SECONDAIRES CLASSIQUES
Sessions 2023 – CORRIGÉ-BARÈME ÉCRIT

| | | | |
|--------------|--|--------------|---------------|
| Date : | 20.09.23 | Durée : | 08:15 - 11:00 |
| Discipline : | Mathématiques - Mathématiques-Analyse | Section(s) : | CC / CC-4LANG |

MATHÉMATIQUES II - Correction

Question 1 (4 points)

Voir EM66 pages 86-87

Question 2 (4 + 6 = 10 points)

1) $6 \cdot (3^x + 3^{-x}) - 7 = 3^{-x+2} \quad D = \mathbb{R}$

$$\Leftrightarrow 6 \cdot 3^x + 6 \cdot 3^{-x} - 7 = 9 \cdot 3^{-x}$$

$$\Leftrightarrow 6 \cdot 3^x + 6 \cdot 3^{-x} - 7 - 9 \cdot 3^{-x} = 0$$

$$\Leftrightarrow 6 \cdot 3^x - 3 \cdot 3^{-x} - 7 = 0 \quad | \cdot 3^x$$

$$\Leftrightarrow 6 \cdot 3^{2x} - 7 \cdot 3^x - 3 = 0 \quad \text{Posons : } y = 3^x > 0$$

$$\Leftrightarrow 6y^2 - 7y - 3 = 0 \quad \left[\Delta = 121 > 0, y_1 = \frac{3}{2}, y_2 = -\frac{1}{3} \right]$$

$$\Leftrightarrow y = \frac{3}{2} \text{ ou } y = -\frac{1}{3}$$

$$\Leftrightarrow 3^x = \frac{3}{2} \text{ ou } \underbrace{3^x = -\frac{1}{3}}_{\text{impossible, car } 3^x > 0}$$

$$\Leftrightarrow x = \log_3 \frac{3}{2}$$

$$\Leftrightarrow x = 1 - \log_3 2 \quad S = \{1 - \log_3 2\}$$

2) $\log_{\sqrt{2}}(2x+3) - \log_2(6-x) \geq 1 - \log_{\frac{1}{2}}(1+2x)$

C.E. : (1) $2x+3 > 0$ (2) $6-x > 0$ (3) $1+2x > 0$

$$\Leftrightarrow x > -\frac{3}{2} \quad \Leftrightarrow x < 6 \quad \Leftrightarrow x > -\frac{1}{2}$$

$$D = \left] -\frac{1}{2}; 6 \right[$$

$$\frac{\ln(2x+3)}{\ln \sqrt{2}} - \frac{\ln(6-x)}{\ln 2} \geq 1 - \frac{\ln(1+2x)}{\ln \frac{1}{2}}$$

$$\Leftrightarrow \frac{\ln(2x+3)}{\frac{1}{2} \ln 2} - \frac{\ln(6-x)}{\ln 2} \geq 1 - \frac{\ln(1+2x)}{-\ln 2} \quad | \cdot \ln 2 > 0$$

$$\Leftrightarrow 2 \ln(2x + 3) - \ln(6 - x) \geq \ln 2 + \ln(1 + 2x)$$

$$\Leftrightarrow \ln(2x + 3)^2 \geq \ln 2 + \ln(1 + 2x) + \ln(6 - x)$$

$$\Leftrightarrow \ln(2x + 3)^2 \geq \ln[2(1 + 2x)(6 - x)]$$

$$\Leftrightarrow 4x^2 + 12x + 9 \geq 12 - 2x + 24x - 4x^2$$

$$\Leftrightarrow 8x^2 - 10x - 3 \geq 0 \quad \left[\Delta = 196 > 0, x_1 = -\frac{1}{4}, x_2 = \frac{3}{2} \right]$$

| | | | | |
|------------------|-----------|----------------|---------------|-----------|
| x | $-\infty$ | $-\frac{1}{4}$ | $\frac{3}{2}$ | $+\infty$ |
| $8x^2 - 10x - 3$ | | + | 0 | - |
| | | 0 | + | |

$$S = \left] -\frac{1}{2}; -\frac{1}{4} \right] \cup \left[\frac{3}{2}; 6 \right[$$

Question 3 (3 + 4 + 4 + 3 + 5 = 19 points)

$$f(x) = x(x - 3)e^{1 - \frac{x}{2}}$$

1) $\text{dom } f = \mathbb{R}$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \underbrace{x(x - 3)}_{\rightarrow +\infty} \underbrace{e^{1 - \frac{x}{2}}}_{\rightarrow +\infty} = +\infty \quad \text{pas d'A.H.G.}$$

A.O.G. ?

$$\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{x(x - 3)e^{1 - \frac{x}{2}}}{x} = \lim_{x \rightarrow -\infty} \underbrace{(x - 3)}_{\rightarrow -\infty} \underbrace{e^{1 - \frac{x}{2}}}_{\rightarrow +\infty} = -\infty \quad \text{pas d'A.O.G.}$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \underbrace{x(x - 3)}_{\rightarrow +\infty} \underbrace{e^{1 - \frac{x}{2}}}_{\rightarrow 0} \text{ f.i.}$$

$$= \lim_{x \rightarrow +\infty} \frac{x(x - 3)}{e^{-1 + \frac{x}{2}}}$$

$$\stackrel{H}{=} \lim_{x \rightarrow +\infty} \frac{2x - 3}{\frac{1}{2}e^{-1 + \frac{x}{2}}}$$

$$\stackrel{H}{=} \lim_{x \rightarrow +\infty} \frac{2}{\frac{1}{4}e^{-1 + \frac{x}{2}}}$$

$$= 0$$

A.H.D.: $y = 0$

2) $\text{dom } f' = \text{dom } f$

$$\begin{aligned} f'(x) &= (2x - 3)e^{1-\frac{x}{2}} + (x^2 - 3x)e^{1-\frac{x}{2}} \left(-\frac{1}{2}\right) \\ &= -\frac{1}{2}e^{1-\frac{x}{2}}(-4x + 6 + x^2 - 3x) \\ &= -\frac{1}{2}e^{1-\frac{x}{2}}(x^2 - 7x + 6) \end{aligned}$$

$$f'(x) = 0 \Leftrightarrow x^2 - 7x + 6 = 0 \quad [\Delta = 25 > 0, x_1 = 1, x_2 = 6]$$

$$\Leftrightarrow x = 1 \text{ ou } x = 6$$

Tableau de variation :

| | | | | | | | |
|---------|-----------|---|--------------|-----------|------------------|---|---|
| x | $-\infty$ | 1 | 6 | $+\infty$ | | | |
| $f'(x)$ | | - | 0 | + | 0 | - | |
| $f(x)$ | $+\infty$ | | $-2\sqrt{e}$ | | $\frac{18}{e^2}$ | | 0 |

Minimum : $f(1) = -2\sqrt{e}$

$m(1; -2\sqrt{e})$

Maximum : $f(6) = \frac{18}{e^2}$

$M\left(6; \frac{18}{e^2}\right)$

3) $\text{dom } f'' = \text{dom } f'$

$$\begin{aligned} f''(x) &= \frac{1}{4}e^{1-\frac{x}{2}}(x^2 - 7x + 6) - \frac{1}{2}e^{1-\frac{x}{2}}(2x - 7) \\ &= \frac{1}{4}e^{1-\frac{x}{2}}(x^2 - 7x + 6 - 4x + 14) \\ &= \frac{1}{4}e^{1-\frac{x}{2}}(x^2 - 11x + 20) \end{aligned}$$

$$f''(x) = 0 \Leftrightarrow x^2 - 11x + 20 = 0 \quad \left[\Delta = 41 > 0, x_1 = \frac{11 + \sqrt{41}}{2}, x_2 = \frac{11 - \sqrt{41}}{2} \right]$$

$$\Leftrightarrow x = \frac{11 + \sqrt{41}}{2} \text{ ou } x = \frac{11 - \sqrt{41}}{2}$$

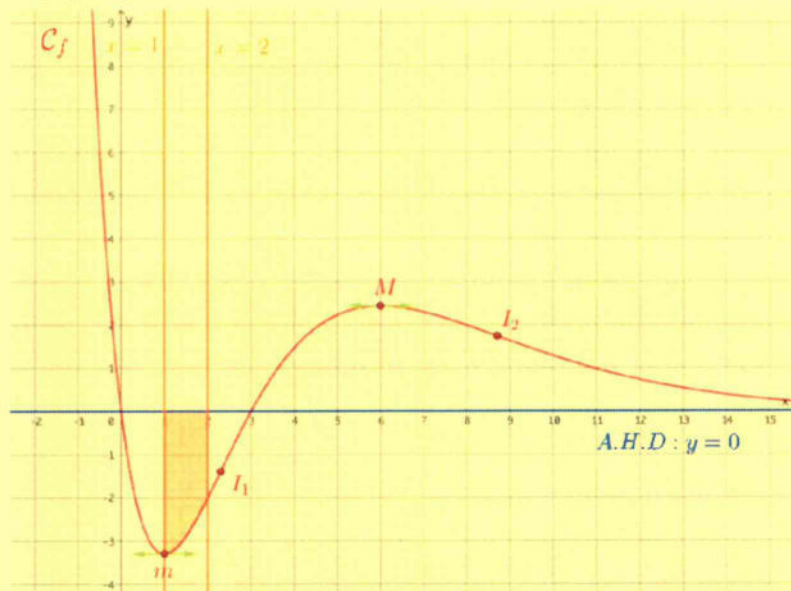
Tableau de concavité :

| | | | | | | |
|-----------------|-----------|----------------------------|----------------------------|-----------|------|----------|
| x | $-\infty$ | $\frac{11 - \sqrt{41}}{2}$ | $\frac{11 + \sqrt{41}}{2}$ | $+\infty$ | | |
| $f''(x)$ | | + | 0 | - | 0 | + |
| \mathcal{C}_f | | \smile | P.I. | \frown | P.I. | \smile |

$$\text{Points d'inflexion : } f\left(\frac{11 - \sqrt{41}}{2}\right) \approx -1,39 \quad I_1 (\approx 2,30; \approx -1,39)$$

$$f\left(\frac{11 + \sqrt{41}}{2}\right) \approx 1,74 \quad I_2 (\approx 8,7; \approx 1,74)$$

4) Représentation graphique :



5) Aire :

$$A = - \int_1^2 (x^2 - 3x)e^{1-\frac{x}{2}} dx \quad f(x) = x^2 - 3x \quad g'(x) = e^{1-\frac{x}{2}}$$

$$f'(x) = 2x - 3 \quad g(x) = -2e^{1-\frac{x}{2}}$$

$$\stackrel{IPP}{=} [2(x^2 - 3x)e^{1-\frac{x}{2}}]_1^2 + \int_1^2 (-4x + 6)e^{1-\frac{x}{2}} dx$$

$$f(x) = -4x + 6 \quad g'(x) = e^{1-\frac{x}{2}}$$

$$f'(x) = -4 \quad g(x) = -2e^{1-\frac{x}{2}}$$

$$\stackrel{IPP}{=} -4 + 4\sqrt{e} + [-2(-4x + 6)e^{1-\frac{x}{2}}]_1^2 - \int_1^2 8e^{1-\frac{x}{2}} dx$$

$$= -4 + 4\sqrt{e} + 4 + 4\sqrt{e} + [16e^{1-\frac{x}{2}}]_1^2$$

$$= 8\sqrt{e} + 16 - 16\sqrt{e}$$

$$= 16 - 8\sqrt{e}$$

$$= 8(2 - \sqrt{e}) \approx 2,81 \text{ u.a.}$$

Question 4 (6 + 3 = 9 points)

$$f(x) = x + 2 - \ln \frac{2x}{x+1}$$

1) C.E. (1) $\frac{2x}{x+1} > 0$

(2) $x + 1 \neq 0$

$\Leftrightarrow x \neq -1$

| | | | | | | |
|------------------|-----------|------|------|-----------|-----|-----|
| x | $-\infty$ | -1 | 0 | $+\infty$ | | |
| $2x$ | | $-$ | $-$ | 0 | $+$ | |
| $x+1$ | | $-$ | 0 | $+$ | $+$ | |
| $\frac{2x}{x+1}$ | | $+$ | $ $ | $-$ | 0 | $+$ |

$dom f =]-\infty; -1[\cup]0; +\infty[$

$$\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \left(\underbrace{x+2}_{\rightarrow \pm\infty} - \underbrace{\ln \frac{2x}{x+1}}_{\rightarrow \ln 2(*)} \right) = \pm\infty$$

pas d'A.H.G.

(*) $\lim_{x \rightarrow \pm\infty} \frac{2x}{x+1} \stackrel{H}{=} \lim_{x \rightarrow \pm\infty} \frac{2x}{x} = 2$

A.O. ?

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} &= \lim_{x \rightarrow \pm\infty} \frac{x+2 - \ln \frac{2x}{x+1}}{x} \\ &= \lim_{x \rightarrow \pm\infty} \left(1 + \underbrace{\frac{2}{x}}_{\rightarrow 0} - \underbrace{\frac{1}{x}}_{\rightarrow 0} \cdot \underbrace{\ln \frac{2x}{x+1}}_{\rightarrow \ln 2} \right) \\ &= 1 (= a) \end{aligned}$$

$$\lim_{x \rightarrow \pm\infty} [f(x) - x] = \lim_{x \rightarrow \pm\infty} \left(2 - \underbrace{\ln \frac{2x}{x+1}}_{\rightarrow \ln 2} \right) = 2 - \ln 2$$

Donc : A.O. : $y = x + 2 - \ln 2$

$$\lim_{x \rightarrow (-1)^-} f(x) = \lim_{x \rightarrow (-1)^-} \left(\underbrace{x+2}_{\rightarrow 1} - \underbrace{\ln \frac{2x}{x+1}}_{\rightarrow +\infty} \right) = -\infty$$

A.V. : $x = -1$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left(\underbrace{x+2}_{\rightarrow 2} - \underbrace{\ln \frac{2x}{x+1}}_{\rightarrow -\infty} \right) = +\infty$$

A.V. : $x = 0$

2) $\forall x \in]-\infty; -1[\cup]0; +\infty[: \varphi(x) = f(x) - (x + 2 - \ln 2) = -\ln \frac{2x}{x+1} + \ln 2$

Réolvons : $-\ln \frac{2x}{x+1} + \ln 2 > 0 \Leftrightarrow \ln \frac{2x}{x+1} < \ln 2 \Leftrightarrow \frac{-2}{x+1} < 0 \Leftrightarrow \frac{1}{x+1} > 0 \Leftrightarrow x > -1$

| | | | | |
|--------------|------------|-------------|-------------|------------|
| x | $-\infty$ | -1 | 0 | $+\infty$ |
| $\varphi(x)$ | $-$ | \parallel | \parallel | $+$ |
| Position | $A.O./C_f$ | \parallel | \parallel | $C_f/A.O.$ |

Question 5 (3 + 3 = 6 points)

1) $\lim_{x \rightarrow -\infty} \left(1 - \frac{3}{x}\right)^{1-2x} = \lim_{h \rightarrow 0^+} (1+h)^{1+\frac{6}{h}}$ Posons : $h = -\frac{3}{x} \Leftrightarrow x = -\frac{3}{h}$
 $= \lim_{h \rightarrow 0^+} \left[(1+h)^{\frac{6}{h}} \cdot (1+h)\right]$ Si $x \rightarrow -\infty$, alors $h \rightarrow 0^+$
 $= e^6$

2) $\int \tan x(1 + \sin 2x) dx = \int (\tan x + 2 \sin x \cos x \tan x) dx$
 $= \int \left(\frac{\sin x}{\cos x} + 2 \sin^2 x\right) dx$
 $= \int \left(-\frac{\sin x}{\cos x} + 1 - \frac{1}{2} \cdot \cos 2x \cdot 2\right) dx$
 $= -\ln |\cos x| + x - \frac{1}{2} \sin 2x + c \quad (c \in \mathbb{R})$

Question 6 (3 + 4 = 7 points)

1) $f(x) = \frac{5x^2 - 7x - 7}{(x^2 + 9)(2x - 1)}$ $dom f = \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$

$$\frac{5x^2 - 7x - 7}{(x^2 + 9)(2x - 1)} = \frac{ax + b}{x^2 + 9} + \frac{c}{2x - 1} \quad \forall x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$$

$$\Leftrightarrow \frac{5x^2 - 7x - 7}{(x^2 + 9)(2x - 1)} = \frac{(ax + b)(2x - 1) + c(x^2 + 9)}{(x^2 + 9)(2x - 1)} \quad \forall x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$$

$$\Leftrightarrow 5x^2 - 7x - 7 = 2ax^2 - ax + 2bx - b + cx^2 + 9c \quad \forall x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$$

$$\Leftrightarrow 5x^2 - 7x - 7 = (2a + c)x^2 + (-a + 2b)x - b + 9c \quad \forall x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$$

$$\Leftrightarrow \begin{cases} 2a + c = 5 \\ -a + 2b = -7 \\ -b + 9c = -7 \end{cases} \Leftrightarrow \begin{cases} c = 5 - 2a \\ a = 2b + 7 \\ -b - 18a = -52 \end{cases} \Leftrightarrow \begin{cases} c = 5 - 2a \\ a = 2b + 7 \\ -37b = 74 \end{cases} \Leftrightarrow \begin{cases} c = -1 \\ a = 3 \\ b = -2 \end{cases}$$

Donc : $f(x) = \frac{3x - 2}{x^2 + 9} - \frac{1}{2x - 1}$

$$\begin{aligned}
 2) \quad f(x) &= \frac{3}{2} \cdot \frac{2x}{x^2+9} - \frac{2}{3} \cdot \frac{1}{\left(\frac{x}{3}\right)^2+1} \cdot \frac{1}{3} - \frac{1}{2} \cdot \frac{1}{2x-1} \\
 F(x) &= \frac{3}{2} \ln|x^2+9| - \frac{2}{3} \operatorname{Arc tan}\left(\frac{x}{3}\right) - \frac{1}{2} \ln|2x-1| + c \quad (c \in \mathbb{R}) \\
 F(0) &= \ln 3 \Leftrightarrow \frac{3}{2} \ln 9 - 0 - \frac{1}{2} \ln 1 + c = \ln 3 \\
 &\Leftrightarrow 3 \ln 3 + c = \ln 3 \\
 &\Leftrightarrow c = -2 \ln 3
 \end{aligned}$$

$$\text{Donc : } F(x) = \frac{3}{2} \ln(x^2+9) - \frac{2}{3} \operatorname{Arc tan}\left(\frac{x}{3}\right) - \frac{1}{2} \ln|2x-1| - 2 \ln 3$$

Question 7 (5 points)

Volume :

$$\begin{aligned}
 V &= \pi \int_{-2}^0 [(f(x))^2 - (g(x))^2] dx \\
 &= \pi \int_{-2}^0 \left[(2^{-x} - 6)^2 - \left(-\frac{3}{2}x - 5\right)^2 \right] dx \\
 &= \pi \int_{-2}^0 \left(2^{-2x} - 12 \cdot 2^{-x} + 36 - \frac{9}{4}x^2 - 15x - 25 \right) dx \\
 &= \pi \int_{-2}^0 \left(2^{-2x} - 12 \cdot 2^{-x} - \frac{9}{4}x^2 - 15x + 11 \right) dx \\
 &= \pi \left[\frac{2^{-2x}}{-2 \ln 2} + 12 \cdot \frac{2^{-x}}{\ln 2} - \frac{3}{4}x^3 - \frac{15}{2}x^2 + 11x \right]_{-2}^0 \\
 &= \pi \left[\left(\frac{1}{-2 \ln 2} + \frac{12}{\ln 2} - 0 - 0 + 0 \right) - \left(\frac{2^4}{-2 \ln 2} + 12 \cdot \frac{2^2}{\ln 2} + 6 - 30 - 22 \right) \right] \\
 &= \pi \left(\frac{1}{-2 \ln 2} + \frac{12}{\ln 2} + \frac{8}{\ln 2} - \frac{48}{\ln 2} + 46 \right) \\
 &= \pi \left(46 - \frac{56}{2 \ln 2} - \frac{1}{2 \ln 2} \right) \\
 &= \pi \left(46 - \frac{57}{2 \ln 2} \right) \approx 15,34 \text{ u.v.}
 \end{aligned}$$